

Halting Planetary Migration

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ABSTRACT

When Jupiter's Roche Lobe radius exceeded the scale height of the protoplanetary disk, Jupiter opened a gap in the disk. When the gap was wide enough, tidal torques from the disk interior and exterior to Jupiter were suppressed and migration continued on the accretion time scale. In the 'minimum solar nebula' about two Jupiter masses of gas remained between Jupiter and Saturn and about five Jupiter masses between Jupiter and Uranus. Unless all but a Jupiter mass of the outer disk was removed, Jupiter would have migrated into the Sun on the accretion time scale. So far, no mechanism has been postulated to remove the outer disk except for photo-evaporation which might work exterior to Saturn. For extra-solar Jupiters at an AU or less from the central star, photo-evaporation would not remove the outer disk.

We propose a new mechanism, relying on the irradiation of the exposed edge of the gap by the Sun, to cause gas from the outer disk to cross the gap and become part of the inner disk, whence it was later accreted by the Sun.

Subject headings: extrasolar planetary systems: formation; disks – radiative transfer; solar system: formation

1. Introduction

Current theories of disk dispersal remove the 'inner disk' (the disk between the Sun and Jupiter) by accretion, and remove the disk exterior to Saturn by photo-evaporation (see Hollenbach, Yorke, & Johnstone 2000). However, even in the minimum solar nebula proposed by Hayashi (1981), about 2 Jupiter masses of material remain between Jupiter and Saturn. After Jupiter carves a gap in the disk, the inner disk accretes onto the Sun,

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and with more than a Jupiter mass exterior to Jupiter, Jupiter follows the inner disk into the Sun. Two detailed numerical simulations (Nelson & Benz 1999; Nelson et al. 1999) reproduce the creation of the gap, but do not follow the further migration of the planet on the accretion time scale.

Even before these simulations, we concluded that to keep Jupiter from migrating into the Sun (on the accretion time scale) we would have to get rid of the disk exterior to Jupiter (the outer disk). We came up with a number of fanciful speculations but were unable to explain why they were not picked up by the simulations. We were driven to read our own recent work on 'The Snow Line' (Sasselov & Lecar 2000, thereafter SL2000) which did contain an effect not included in the above simulations, i.e., the irradiation of the disk by the Sun. In a disk with negligible accretion ($< 10^{-8} M_{\odot} \text{yr}^{-1}$) typical of old T Tauri disks, this irradiation controls the temperature of the disk.

In this paper we suggest that the irradiation of the parent star can control the rate of migration and even halt it altogether in the inner regions ($< 10 \text{ AU}$) of a typical protoplanetary disk.

2. The Irradiation Model

Our model is that of a star surrounded by a flared disk, the same as in SL2000. Our disk has a surface gas mass density $\Sigma = r^{-3/2} \Sigma_0$, with r , the distance from the star (or Sun), in AU and $\Sigma_0 = 10^3 \text{ g cm}^{-2}$, which is a standard minimum-mass solar nebula model. The dust and gas are assumed well mixed. The emergent spectrum of the star is calculated with a stellar model atmosphere code with Kurucz (1992) line lists and opacities. The disk intercepts the stellar radiation $F_{irr}(r)$ at a small grazing angle (typically 3° at 1 AU). After the planet opens a gap, the *wall* at the gap's outer rim is exposed to the star. In this paper we work with a fairly narrow gap (see §3.), which exposes $\sim 1/5$ of the wall to the direct starlight. We do this because our mechanism (§3.) starts working as soon as even a partial gap has been carved out; the much wider gaps seen in simulations by, *e.g.*, Nelson & Benz (1999) are a later phase in the evolution of the disk. We assume the gap wall to be inclined at 60° to the disk plane at this early stage in the development of the gap (a range of 45° - 90° does not make a difference). Thus, the flux intercepted by a unit surface area of the gap's wall can be 5 or more times greater than in the unperturbed disk. In our computation we account for the cooling taking place in the midplane of the gap's outer wall which is in the shadow of the inner disk and the pressure gradient which develops.

We compute the radiative transfer for the gap’s outer rim in the same fashion as for the rest of the disk (for details, see SL2000). In particular, we use dust grains with properties which best describe the disks of T Tauri stars. Our calculation for the gap’s wall is generally valid for $r \geq 0.2$ AU, so that the temperatures never exceed 1500-1800K and we do not consider dust sublimation; the dust is present at all times and is the dominant opacity source. This is required for our radiative transfer solution.

The midplane temperature in our passive unperturbed disk is derived from the requirements of hydrostatic and radiative equilibrium, and the balance between heating by irradiation and radiative cooling, $\sigma T^4(r) = F_{irr}(r)$. Our midplane temperature scales as $T(r) = T_0 r^{-3/7}$ K. The scaling coefficient is $T_0 = 140$; the actual numerical solution for $T(r)$ differs insignificantly from this simple form in the range of interest to us ($0.1 < r < 10$ AU). This result is very insensitive to the surface density, Σ_0 ; we find that the changes in $T(r)$ are within ± 10 K for an increase or decrease in Σ_0 of a factor of 10 (SL2000).

From these calculations here we find, for a gap which is twice the scale height, that the effective photosphere of the gap’s exposed wall is typically about 1.6 times hotter than the corresponding gas temperature at that distance in the unperturbed disk. The ratio is indeed almost constant, slowly varying with r from 0.2 to 5.0 AU. The midplane temperature of the disk returns to the unperturbed temperature within only about 2 gas scale heights behind the gap wall, also a slowly varying function of r . A large pressure gradient develops between the heated part of the wall and the midplane section which is in the shadow.

We note that in a disk with a high accretion rate, irradiation plays no role in its thermal structure even close to the star (see D’Alessio et al. (1999) for such examples). However, planet forming disks may not have such high accretion rates ($> 10^{-8} M_\odot \text{yr}^{-1}$), as evidenced by the large number of passive disks around old T Tauri stars. As in SL2000, we postulate that the latter disks comprise the basis (parameters, dust properties, etc.) of our unperturbed protoplanetary disk model.

3. The Mechanism

We emphasize that the unperturbed disk is razor thin and only slightly flared. The opening angle is $h/r = c/r\Omega = 0.044r^{2/7}$, where h is the scale height of the disk, Ω is the angular frequency ($r\Omega$ is the circular orbital velocity). For the unperturbed disk, before a gap opens, $T(r) = 140r^{-3/7}$ K, so the sound speed is $c = 1.32 \text{ km.s}^{-1} r^{-3/14}$. At Jupiter’s distance, $r_J = 5.203$, $c = 0.821 \text{ km.s}^{-1}$ and $h/r = 0.0705$. Because the disk is so thin, only

a small fraction of the solar radiation is intercepted and absorbed. This fraction is very sensitive to small perturbations in the geometry; hills expose more area and heat up, while valleys, in the shadow, cool down.

We compare this disk with the disk after Jupiter has begun to carve a gap. We focus on an initial gap of radial extent h on either side of Jupiter. The inner disk has $r < r_J - h$ and the outer disk has $r > r_J + h$. The gas in the gap is in the shadow of the inner disk and cools down. Part of the outer disk, formerly in the shadow of the inner disk, is now exposed. We call this the 'wall'. The height of the wall is:

$$\Delta h = h(r_J + h) - h(r_J - h) = 2h \frac{dh}{dr},$$

where

$$\frac{\Delta h}{r} = 0.114 r^{\frac{2}{7}} \frac{h}{r} = 0.182 \frac{h}{r}$$

at r_J . The calculation described in §2 is now applied to this wall and determines that the temperature at the wall increases by a factor of 1.6, and that this higher temperature extends for a distance $\sim 2h$ behind the wall.

The 'wall' increases the area directly exposed to sunlight. The Sun's luminosity, L_\odot , can be expressed in units of the orbital kinetic energy of Jupiter ($E_J = \frac{1}{2} M_J v_J^2 = 1.6 \times 10^{42}$ ergs) and the orbital period of Jupiter ($T_J = 11.9$ years). In those units, $L_\odot = 3.826 \times 10^{33}$ ergs.s $^{-1}$ = $0.9 E_J / T_J$. The luminosity intercepted by the wall is $L_{\text{int}} = 0.013 L_\odot$, so enough energy is intercepted to move a Jupiter mass of gas an appreciable distance in about 80 Jupiter periods.

The mass transfer across the wall is:

$$\frac{dm}{dt} = 2\pi \Sigma r c = 1.20 \times 10^{29} \text{ g/yr} = 0.06 M_J / \text{yr} = 0.75 M_J / T_J,$$

where M_J is the Jupiter mass. At the sound speed, the time to cross the gap, $tc = 2h/c \cong 3 \text{ yrs} \approx \frac{1}{4} T_J$. The energy transferred across the wall is $dE/dt = \frac{1}{2} c^2 dm/dt = 0.013 M_J / T_J \approx L_{\text{int}}$.

Looking at the dynamics in somewhat more detail, in the unperturbed disk, on a hydrodynamic time-scale (i.e., neglecting accretion),

$$\frac{dv}{dt} = -\frac{GM_\odot}{r^2} + r\Omega^2 + \left(-\frac{1}{\rho} \frac{dP}{dr}\right) = 0,$$

where ρ is the gas density and P is the pressure. The zeroth order ($r = r_0$, $\frac{dr}{dt} = 0$) gives us, for the acceleration due to the pressure gradient:

$$g \equiv -\frac{1}{\rho} \frac{dP}{dr} \cong \frac{c^2}{r}.$$

A calculation yields the numerical factor 1.07. The ratio of g to $r\Omega^2$ is $(c/r\Omega)^2 = (h/r)^2$, so this term can be ignored. The first order gives us:

$$g \cong \frac{c^2}{h} \cong c\Omega_0 \cong \left(\frac{\Delta T}{T}\right)^{\frac{1}{2}} c_0 \Omega_0,$$

where we have $r = r_0(1 + x)$ and $\ddot{x} = -\Omega_0^2 x + g$, and from § 2 we have $\Delta T/T = 1.6$.

After the 'wall' is exposed:

$$g = \frac{c^2}{h} = c\Omega,$$

which is an increase by a factor of r/h , and in addition c increases by $(1.6)^{1/2}$. At this point, after the gap, $g = c\Omega$ is comparable to the 'restoring force' at the maximum extension, er :

$$er\Omega^2 = eV\Omega = c\Omega.$$

Note that in this process we conserve angular momentum. Denoting the angular momentum per unit mass by L :

$$L^2 = GMa(1 - e^2) = GMq(1 + e),$$

where $q = a(1 - e)$ is the perihelion distance and e is the eccentricity. *Conserving* angular momentum, $L = \text{const.}$, we find $dq/q = -de/(1 + e) \approx -e$, while the increase in the semi-major axis is insignificant, $da/a = d(e^2)/(1 - e^2) \sim 0$, for small e .

At this point we have described a mechanism by which gas can cross the gap whence it will continue to accrete onto the central star. The gas which crossed the gap, on a sound-speed time scale, will have to be replenished on an accretional time scale. But, where without this mechanism, Jupiter would have been driven into the Sun by the inwardly moving outer disk, we now allow a substantial fraction of the outer disk to cross the gap into the inner disk.

We further suggest, admittedly in a more speculative vein, a mechanism that might speed up the replenishment. Previous work (Lin & Papaloizou 1980, Goldreich & Tremaine 1980; and more recently, Bryden et al. 1999, Kley 1999) has shown that Jupiter exerts tidal torques on it's outer disk and induces trailing density enhancements in a tightly wrapped spiral. This series of ridges (which intercept sunlight directly) and valleys (which cool down because they lie in the shadow of the ridges) initiates our mechanism at every ridge. In effect, they act like a closely spaced grating of mini-gaps and our mechanism will transfer material across each gap. We re-emphasize the fact that the intercepted sunlight depends on h/r which is a slowly increasing function of r (increasing as $r^{2/7}$). The intercepted flux does *not* fall off as r^{-2} , because the area of the intercepting annulus increases as r^2 .

This is as far as these estimates should be taken. Further details should be determined from a numerical simulation. We suggest that such a simulation could be performed using a non-axisymmetric thin disk model, including Jupiter, but with the solar heat input at the wall included. All the previous simulations, that we are aware of, were accretionally heated, and so ignored this effect. We predict that a simulation including this effect will show a natural stop to the migration at substantial distances from the sun.

4. Summary

Unless the outer disk (e.g., beyond Jupiter) is removed, migration on the accretion time scale cannot be halted by any mechanism so far discussed in the literature. We propose a mechanism to halt migration in the inner regions of protoplanetary disks in which irradiation by the central star dominates over accretion. Once a gap is partially open, stellar irradiation provides sufficient energy to the outer wall of the gap to move material across the gap. The mechanism acts as a semi-permeable membrane: material moved outwards returns to the rim of the gap, but material moved inwards, across the gap, never returns. Any planet which opens a gap should trigger the mechanism.

We are grateful to John Papaloizou for useful discussions, and to the referee for thoughtful and helpful comments. DDS acknowledges support from the Alfred P. Sloan Foundation.

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